Small amplitude ion-acoustic double layers in multicomponent plasma with positrons

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Ion-acoustic double layers has been studied in multicomponent plasma with positrons. Using the reductive perturbation method, the modified Korteweg–de Vries (mKdV) equation is derived for the system. The double-layer solution of the mKdV equation is discussed in detail. It is found that there exist two critical concentrations of positrons, α_R and α_Q, which decide the existence and nature of the ion-acoustic double layers. It is also found that the system supports ion-acoustic double layers only when the positron concentration (α) is less than the critical concentration α_R (i.e., $\alpha < \alpha_R$). It is also investigated that for the given set of parameter values, if $\alpha_R < \alpha_Q$, the system supports only rarefactive double layers for the values of α lying in the range $0 < \alpha < \alpha_R$. However, for the given set of parameter values $\alpha_R > \alpha_Q$, the system supports rarefactive double layers for $\alpha < \alpha_Q$, and for $\alpha > \alpha_Q$, compressive double layers exist. The present theory also predicts that for a given set of parameter values on increasing the positron concentration, the amplitude of the rarefactive (compressive) double layer decreases (increases), whereas as positron concentration is increased, the width of the rarefactive (compressive) double layer increases (decreases). The effects of positron concentration and temperature ratio on the characteristics of the double layers (namely amplitude and width) are discussed in detail.

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I. INTRODUCTION

In recent years, there has been considerable interest in understanding the behavior of plasmas consisting of electrons and positrons. The electron-positron plasmas are thought to be generated naturally by pair production in high-energy processes occurring in many astrophysical environments such as the early universe, neutron stars, active galactic nuclei, or pulsar magnetosphere [1–7]. The electron-positron plasmas have also been created in the laboratory [8–10] and encountered in laboratory experiments where positrons are used as probes to study transport in tokamas. Recently, Greaves, Trinkle, and Surko [11] have reported that advances in the positron trapping technique have led to room-temperature plasmas of 10^7 positrons with lifetime 10^3 s. Because of the long lifetime of the positrons most of the astrophysical [7,12] and laboratory plasmas [13–15] become an admixture of electrons, positrons, and ions. In a recent paper, Surko and Murphy [16] have reported that over a wide range of parameters, annihilation of electrons and positrons, which is the analog of recombination in plasma composed of ions and electrons, is relatively unimportant. They have also reported that even at an electron density of 1×10^{12} cm^{-3} and a temperature as low as 1 eV, the positron annihilation time is greater than 1 s. Therefore, the study of electron-positron-ion (EPI) plasmas is important to understand the behavior of both astrophysical and laboratory plasmas.

During the last few years, the nonlinear waves propagating in EPI plasmas have received a great deal of attention in investigating the nonlinear structures. The envelope solitons associated with the electromagnetic waves in EPI plasmas have been studied by Rizzato [17] and Benezhiani *et al.* [18]. Ion-acoustic solitons in EPI plasmas have been studied by Popel *et al.* [19]. Neijoh [20] studied the effect of the ion temperature on large amplitude ion-acoustic waves in an EPI plasma. The double layer associated with the kinetic Alfven waves in a magnetized electron-positron-ion plasma has been studied by Kakatni and Goswami [21]. Recently, the nonlinear propagation of ion-acoustic waves in electron-positron plasma with trapped electrons has been studied by Alinejad and Sobhanea [22]. In a recent paper [23] we have studied the effects of positron density and temperature on ion-acoustic dressed solitons in electron-positron-ion plasma. However, ion-acoustic double layers in electron-positron-ion plasma have not been studied so far.

The study of double layers in plasmas attracted much attention during the last few years due to its relevance in cosmic applications [24–26], confinement of plasma in tandem mirror devices [27], for ion-heating in linear turbulence heating devices [28], etc. The ion-acoustic double layers in plasmas have been extensively studied experimentally [29–32], theoretically [33–38], and also in numerical simulation experiments [39,40]. Double layers occur naturally in a variety of space plasma environments and are of great interest in astrophysics [41]. The Viking satellite [42] observed the double layers in the magnetospheric regions. The measurements of the S3-3 satellite [25] also show the existence of double layers structures in the auroral regions. It is well known that an electric double layer is a region within plasma in which localized charged regions can sustain a potential difference and cause electron and ion acceleration [36]. In addition to this, the double layers have applications in space propulsion [43,44] and in modeling of astrophysical processes [45]. Several theoretical models have been developed to explain double layers. Among them, the modified Korteweg–de Vries (mKdV) equation with cubic nonlinearity has also been widely employed to describe the propagation characteristics of electrostatic double layers in different plasma systems. Using the reductive perturbation method several authors [35,37,38] have derived the mKdV equation

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and studied weak ion-acoustic double layers in different plasma systems.

Two-electron temperature distributions are very common in the laboratory [31,46], as well as in space plasmas [25]. The ion-acoustic double layers in the two-electron temperature plasma have been investigated extensively, theoretically [37,38] as well as experimentally [31,32]. Ion-acoustic double layers have also been observed in the auroral and magnetosphere plasmas, where the two-electron species exist [25,42]. In a recent paper, Shatashvili *et al.* [47] have reported that out flows of electron-positron plasma from pulsars entering an interstellar cold, low-density electron-ion plasma form two-temperature electron-positron-ion plasma. Therefore, it is interesting to investigate ion-acoustic double layers in multicomponent plasma, with hot positrons, cold ions, and two-temperature distributions of electrons.

The aim of this paper is to study the ion-acoustic double layers in multicomponent plasma consisting of hot positrons, cold ions, and electrons with two-electron temperature distributions. The salient feature is to demonstrate the existence of compressive and rarefactive ion-acoustic double layers in the electron-positron-ion plasma. Using the reductive perturbation method, the modified Korteweg–de Vries (mKdV) equation is derived for the system. The double-layer solution of the mKdV equation is discussed in detail. It is found that there exist two critical concentrations of positrons, α_R and α_Q, which decide the existence and nature of the ion-acoustic double layers. It is also found that the system supports ion-acoustic double layers only when the positron concentration (α) is less than the critical concentration α_R (i.e., $\alpha<\alpha_R$). It is also investigated that for the given set of parameter values, if $\alpha_R<\alpha_Q$, the system supports only rarefactive double layers for the values of α lying in the range $0<\alpha<\alpha_R$. However, for the given set of parameter values if $\alpha_R>\alpha_Q$, the system supports rarefactive double layers for $\alpha<\alpha_Q$, and for $\alpha>\alpha_Q$, compressive double layers exist. The present theory also predicts that for a given set of parameter values on increasing the positron concentration, the amplitude of the rarefactive (compressive) double layer decreases (increases), whereas as positron concentration is increased, the width of the rarefactive (compressive) double layer increases (decreases). The effects of positron concentration and temperature ratio on the characteristics of the double layers (namely amplitude and width) are discussed in detail.

The paper has been organized as follows: In Sec. II, the normalized fluid equations for the system have been presented. Using the reductive perturbation method, the mKdV equation has been derived in Sec. III and in Sec. IV, the double layer solution of the mKdV equation has been obtained. Discussion is given in Sec. V and the main conclusions have been summarized in Sec. VI.

II. BASIC EQUATIONS

We consider a multicomponent plasma consisting of hot positrons, cold ions, and electrons, which are divided into two groups: a hot component with density n_h and temperature T_h and a cold component with density n_c and temperature T_c. The nonlinear behavior of ion-acoustic waves may be described by the following set of normalized fluid equations:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nv)=0, \tag{1}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=-\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_h+n_c-\alpha n_p-(1-\alpha)n, \tag{3}$$

$$n_h=\nu \exp\left(\frac{\beta}{(\mu+\nu\beta)}\phi\right), \tag{4}$$

$$n_c=\mu \exp\left(\frac{1}{(\mu+\nu\beta)}\phi\right), \tag{5}$$

$$n_p=\exp(-\gamma\phi). \tag{6}$$

For small ϕ, from Eqs. (4) and (5), we have

$$n_e=n_h+n_c=1+\phi+\frac{1}{2}\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\phi^2+\frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}\phi^3+\cdots, \tag{7}$$

where

$$\alpha=\frac{n_{p0}}{n_0},\quad \mu=\frac{n_{c0}}{n_0},\quad \nu=\frac{n_{h0}}{n_0},\quad \beta=\frac{T_c}{T_h},\quad \gamma=\frac{T_{eff}}{T_p}.$$

It may be noted that in the present case we have assumed that the positron annihilation time is larger than the inverse of the characteristic frequency of the ion-acoustic wave. Under such conditions, one can assume that annihilation of positrons with electrons is negligible and the effect of annihilation can be neglected. In the above equations, n and v are the density and fluid velocity of the ion species, n_p is the positron density, n_{c0}, n_{h0}, n_{p0}, and n_0 are the equilibrium densities of two-electron components, positron component and ion component, respectively, ϕ is the electrostatic potential, and α is the equilibrium density ratio of positron to electron species.

In Eqs. (4)–(6), the electron and positron density distributions are considered to be of the Maxwell-Boltzmann type. In the above equations (1)–(6), velocity v, potential (ϕ), time (t), and space coordinate (x) have been normalized with respect to ion-acoustic speed C_s, thermal potential $\frac{T_{eff}}{e}$, inverse of the ion-plasma frequency ω_{pi}^{-1}, and Debye length λ_D, respectively. Ion density n and positron density n_p are normalized with their corresponding equilibrium values, whereas electron densities n_h and n_c are normalized by n_0. The ion-acoustic speed C_s, the ion plasma frequency ω_{pi}, and the Debye length λ_D are given by

$$C_s=\left(\frac{T_{eff}}{M}\right)^{1/2},\quad \omega_{pi}=\left(\frac{4\pi n_e^{(0)}e^2}{M}\right)^{1/2},$$

$$\lambda_D=\left[\frac{T_{eff}}{4\pi n_e^{(0)}e^2}\right]^{1/2}.$$

The charge-neutrality condition is expressed as $\mu+\nu=1$.

III. DERIVATION OF THE mKdV EQUATION

To derive the mKdV equation from the basic set of equations, viz. Eqs. (1)–(6), we introduce the following stretching of coordinates (ξ) and (τ) as

$$\xi=\varepsilon(x-St) \tag{8a}$$

and

$$\tau=\varepsilon^3 t, \tag{8b}$$

where ε is a smallness parameter and S is the phase velocity of the wave, to be determined later.

Now we expand the field quantities in the following form

$$\begin{bmatrix} n \\ v \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_1 \\ v_1 \\ \phi_1 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_2 \\ v_2 \\ \phi_2 \end{bmatrix} + \varepsilon^3 \begin{bmatrix} n_3 \\ v_3 \\ \phi_3 \end{bmatrix} \cdots . \tag{9}$$

On substituting the expansion (9) into Eqs. (1)–(6), using Eqs. (8a) and (8b), and equating terms with the same powers of ε, we obtain a set of equations for each order in ε. The set of equations at the lowest order, i.e., $O(\varepsilon^2)$ of continuity equation (1) and momentum equation (2) and $O(\varepsilon)$ of the Poisson's equation (3), are

$$-S\frac{\partial n_1}{\partial \xi}+\frac{\partial v_1}{\partial \xi}=0, \tag{10}$$

$$-S\frac{\partial v_1}{\partial \xi}+\frac{\partial \phi_1}{\partial \xi}=0, \tag{11}$$

$$(1+\alpha\gamma)\phi_1-(1-\alpha)n_1=0. \tag{12}$$

Now substituting Eqs. (10)–(12) and using the condition that the perturbed quantities (n_1,v_1,ϕ_1) tend to zero as $\xi\to\pm\infty$, we obtain the following first-order solutions:

$$n_1=\frac{1}{S^2}\phi_1, \tag{13}$$

$$v_1=\frac{1}{S}\phi_1. \tag{14}$$

On using Eqs. (13) and (14) in the Poisson equation (12), we obtain the following linear dispersion relation:

$$S=\left(\frac{(1-\alpha)}{(1+\alpha\gamma)}\right)^{1/2}. \tag{15}$$

Now taking the next higher order, i.e., $O(\varepsilon^3)$ of continuity equation (1) and momentum equation (2) and $O(\varepsilon^2)$ of the Poisson's equation (3) are

$$-S\frac{\partial n_2}{\partial \xi}+\frac{\partial}{\partial \xi}(n_1v_1)+\frac{\partial v_2}{\partial \xi}=0, \tag{16}$$

$$-S\frac{\partial v_2}{\partial \xi}+v_1\frac{\partial v_1}{\partial \xi}+\frac{\partial \phi_2}{\partial \xi}=0, \tag{17}$$

$$-(1+\alpha\gamma)\phi_2-\frac{1}{2}(1-\alpha\gamma^2)\phi_1^2+(1-\alpha)n_2=0. \tag{18}$$

Now using the first-order solutions, Eqs. (13) and (14), we obtain the following second-order solutions:

$$n_2=\frac{1}{S^2}\left[\frac{3}{2S^2}\phi_1^2+\phi_2\right], \tag{19}$$

$$v_2=\frac{1}{2S^3}\phi_1^2+\frac{1}{S}\phi_2. \tag{20}$$

The Poisson equation (3) at $O(\varepsilon^2)$ gives

$$Q\phi_1^2=0 \tag{21}$$

where

$$Q=\frac{3}{2}\frac{(1-\alpha)}{S^4}-\frac{1}{2}(1-\alpha\gamma^2)-\frac{1}{2}\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}. \tag{22}$$

Since $\phi_1\neq 0$, "Q" should be at least of the order of ε and now $Q{\phi_1}^{2n}$ becomes of the order of ε^3; so it should be included in the next order of Poisson's equation.

The next higher order, i.e., $O(\varepsilon^4)$ of the continuity equation and momentum equation and $O(\varepsilon^3)$ of the Poisson equation, are

$$-S\frac{\partial n_3}{\partial \xi}+\frac{\partial n_1}{\partial \tau}+\frac{\partial}{\partial \xi}(n_2v_1)+\frac{\partial}{\partial \xi}(n_1v_2)+\frac{\partial v_3}{\partial \xi}=0, \tag{23}$$

$$-S\frac{\partial v_3}{\partial \xi}+\frac{\partial v_1}{\partial \tau}+v_1\frac{\partial v_2}{\partial \xi}+v_2\frac{\partial v_1}{\partial \xi}+\frac{\partial \phi_3}{\partial \xi}=0, \tag{24}$$

$$\frac{\partial^2\phi_1}{\partial \xi^2}-(1+\alpha\gamma)\phi_3-(1-\alpha\gamma^2)\phi_1\phi_2-\frac{1}{6}(1+\alpha\gamma^3)\phi_1^3 +(1-\alpha)n_3=0. \tag{25}$$

On using Eqs. (23) and (24) and first- and second-order solutions in Eq. (25), we obtain the following mKdV equation:

$$P\frac{\partial \phi}{\partial \tau}+Q\frac{\partial \phi^2}{\partial \xi}+R\frac{\partial \phi^3}{\partial \xi}+\frac{\partial^3\phi}{\partial \xi^3}=0, \tag{26}$$

where

$$P=\frac{2(1-\alpha)}{S^3} \tag{27}$$

and

$$R=\frac{3}{2}\frac{(1-\alpha)}{S^6}-\frac{(1+\alpha\gamma^3)}{6}-\frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}. \tag{28}$$

In Eq. (28), ϕ is used in place of $\phi^{(1)}$ for brevity.

IV. DOUBLE LAYER SOLUTION

For the steady-state solution of the mKdV equation (26), we use the transformation

$$\eta = \xi - u\tau, \tag{29}$$

where u is a constant velocity. Using Eq. (20) in Eq. (17) and integrating with respect to η, we obtain

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 + V(\phi) = 0, \tag{30}$$

where $V(\phi)$ is the Sagdeev potential given by

$$V(\phi) = -\frac{1}{2}Pu\phi^2 + \frac{1}{3}Q\phi^3 + \frac{1}{4}R\phi^4. \tag{31}$$

In the derivation of Eq. (21), we have used the following boundary conditions:

$$\text{As } \phi \to 0, \quad \frac{d\phi}{d\eta} \text{ and } \frac{d^2\phi}{d\eta^2} \to 0.$$

However, for the double-layer solution, the Sagdeev potential should be negative between $\phi=0$ and ϕ_m, where ϕ_m is some extremum value of the potential.

In order that the Sagdeev potential $V(\phi)$ gives rise to a double-layer solution, it should satisfy the following conditions:

$$V(\phi) = 0, \quad \text{at } \phi = 0 \text{ and } \phi = \phi_m, \tag{32a}$$

$$V'(\phi) = 0, \quad \text{at } \phi = 0 \text{ and } \phi = \phi_m, \tag{32b}$$

$$V''(\phi) < 0, \quad \text{at } \phi = 0 \text{ and } \phi = \phi_m. \tag{32c}$$

Applying the boundary condition (32a) and (32b) in Eq. (31), we obtain

$$u = \left(-\frac{R}{2P}\right)\phi_m^2 \tag{33}$$

and

$$\phi_m = -\frac{2}{3}\frac{Q}{R} = -\frac{1}{3}\left(\frac{3(1-\alpha)}{S^4} - (1-\alpha\gamma^2) - \frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\right) \times \left(\frac{3}{2}\frac{(1-\alpha)}{S^6} - \frac{(1+\alpha\gamma^3)}{6} - \frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}\right)^{-1}. \tag{34}$$

Using Eqs. (33) and (34) in Eq. (31), we obtain

$$\begin{aligned} V(\phi) &= \frac{R\phi^2}{4}(\phi_m - \phi)^2 \\ &= \frac{\phi^2}{4}\left[\frac{3}{2}\frac{(1-\alpha)}{S^6} - \frac{(1+\alpha\gamma^3)}{6} - \frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}\right] \\ &\quad \times \left[-\frac{1}{3}\left(\frac{3(1-\alpha)}{S^4} - (1-\alpha\gamma^2) - \frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\right)\right. \\ &\quad \left.\times \left\{\frac{3}{2}\frac{(1-\alpha)}{S^6} - \frac{(1+\alpha\gamma^3)}{6} - \frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}\right\}^{-1} - \phi\right]^2. \end{aligned} \tag{35}$$

The double-layer solution of Eq. (30) with Eq. (35) is given by

$$\phi = \frac{\phi_m}{2}\left[1 - \tanh\left\{\left(-\frac{R}{8}\right)^{1/2}\phi_m(\xi - u\tau)\right\}\right]. \tag{36}$$

It may be noted from the above equation that for the existence of a double layer, the coefficient of the cubic nonlinear term of the mKdV equation, i.e., R, should be negative. It may also be noted from Eq. (34) that the nature of the double layer, i.e., whether the system will support a compressive or a rarefactive double layer, depends upon the sign of Q. If Q is positive, a compressive double layer exists, whereas for negative Q, a rarefactive double layer exists.

The thickness W of the double layer is given by

$$W = \frac{2\left(-\frac{8}{R}\right)^{1/2}}{|\phi_m|} = 2\left(-\frac{8}{\left(\frac{3}{2}\frac{(1-\alpha)}{S^6} - \frac{(1+\alpha\gamma^3)}{6} - \frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}\right)}\right)^{1/2} \times \left|-\frac{1}{3}\left(\frac{3(1-\alpha)}{S^4} - (1-\alpha\gamma^2) - \frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\right)\left(\frac{3}{2}\frac{(1-\alpha)}{S^6} - \frac{(1+\alpha\gamma^3)}{6} - \frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}\right)^{-1}\right|^{-1}. \tag{37}$$

V. DISCUSSION

To investigate the existence regions and nature of the ion-acoustic double layers in electron-positron-ion plasma, we have done numerical calculations for different set of parameters (μ, β, and γ). Our investigations show that the presence of positron species drastically affects the existence regions and nature of the ion-acoustic double layers. Equations (22) and (28) show that the coefficients Q and R are functions of the positron concentration α. For a given set of parameters (μ, β, and γ) the coefficient R becomes positive as α exceeds the critical concentration α_R, hence the system may support ion-acoustic double layers. Moreover, for the existence of a weak double layer the coefficient Q must be of the order of ε. It may be noted that a double layer can form in the electron-positron-ion plasma system only with appropriate values of the physical parameters.

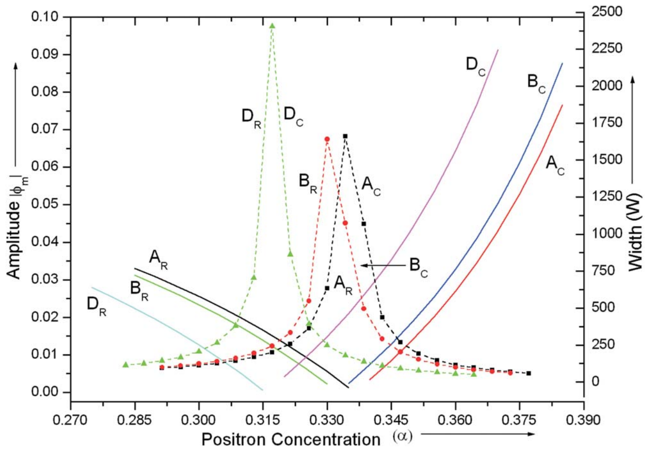

FIG. 1. (Color online) Variation of the amplitude $|\phi_m|$ (shown by solid curve) and width W (shown by dashed curve) of the ion-acoustic double layers with positron concentration (α) with $\mu=0.01$ and $\beta=0.05$ for different values of (γ). Curves A, B, and D refer to $|\phi_m|$ and W for three sets of values of $\gamma=0.001$, 0.01, and 0.05, respectively. The subscripts C and R correspond to compressive and rarefactive double layers, respectively.

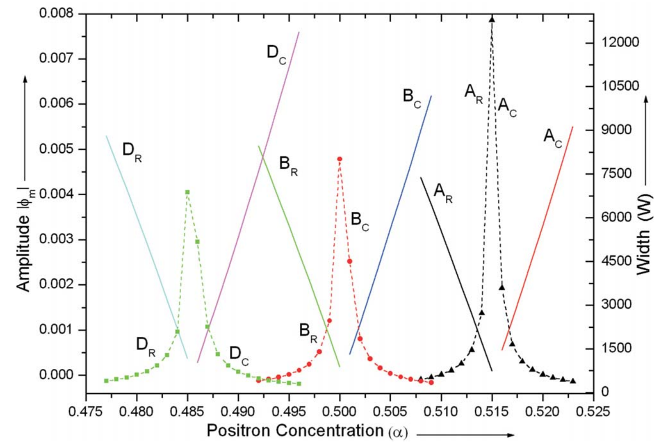

FIG. 3. (Color online) Variation of the amplitude $|\phi_m|$ (shown by solid curve) and width W (shown by dashed curve) of the ion-acoustic double layers with positron concentration (α) with $\gamma=0.1$ and $\mu=0.01$ for different values of (β). Curves A, B, and D refer to $|\phi_m|$ and W for three sets of values of $\beta=0.034$, 0.035, and 0.036, respectively. The subscripts C and R correspond to compressive and rarefactive double layers, respectively.

We have calculated the amplitude and width of the ion-acoustic double layer using Eqs. (34) and (37), respectively. The variation of amplitude $|\phi_m|$ and width W of the double layer with respect to the positron concentration (α) is shown in Fig. 1 for given values of (μ and β) and three different values of γ. In Fig. 1, curve A shows the variation of the amplitude (shown by full line) and the width (shown by the dashed line) of the ion-acoustic double layer with the positron concentration (α), for a set of parameter values ($\mu=0.01$ and $\beta=0.05$) amounting to $\gamma=0.001$. Curves B and D show similar variation for parameter $\gamma=0.01$ and $\gamma=0.05$, respectively. Curves A_R, B_R, and D_R represent the variation of the amplitude (shown by the solid line) and the width (shown by the dashed line) of the rarefactive double layers. For the values of the positron concentration $\alpha<\alpha_R$, as we increase the positron concentration, the amplitude of the rarefactive double layer decreases until α approaches α_Q. If we further increase the value of α, such that $\alpha>\alpha_Q$, then the system supports compressive double layers whose amplitude (shown by solid curves A_C, B_C, and D_C) increases by increasing α. On the other hand the width of the rarefactive double layer (shown by dashed curves A_R, B_R, and D_R) increases by increasing α and as α approach the value α_Q its value increases rapidly, whereas, the width of the compressive double layer (shown by dashed curves A_C, B_C, and D_C) decreases rapidly with increasing α and then becomes nearly constant. These curves show that for a given set of param-

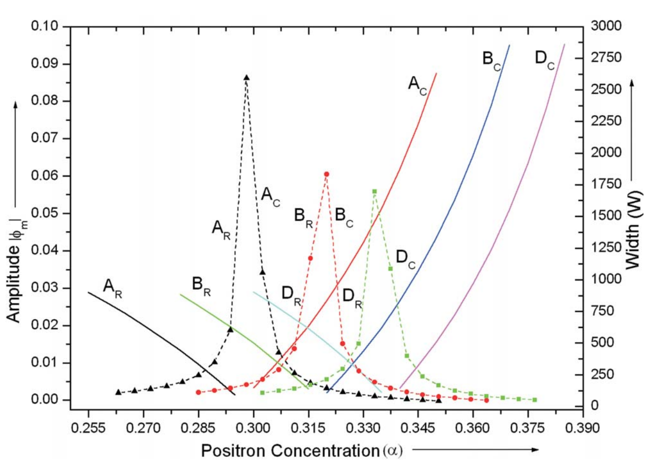

FIG. 2. (Color online) Variation of the amplitude $|\phi_m|$ (shown by solid curve) and width W (shown by dashed curve) of the ion-acoustic double layers with positron concentration (α) with $\gamma=0.1$ and $\beta=0.05$ for different values of (μ). Curves A, B, and D refer to $|\phi_m|$ and W for three sets of values of $\mu=0.01$, 0.011, and 0.012, respectively. The subscripts C and R correspond to compressive and rarefactive double layers, respectively.

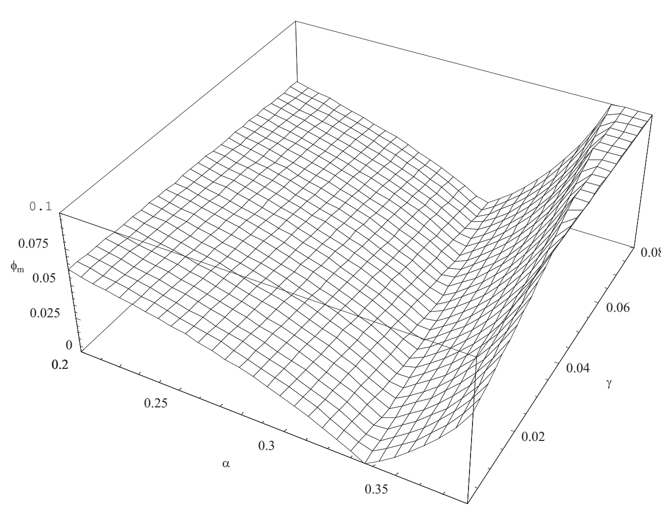

FIG. 4. Three-dimensional plots of the variation of the amplitude $|\phi_m|$ of the ion-acoustic double layers with plasma parameters (α and γ) having $\mu=0.01$ and $\beta=0.05$.

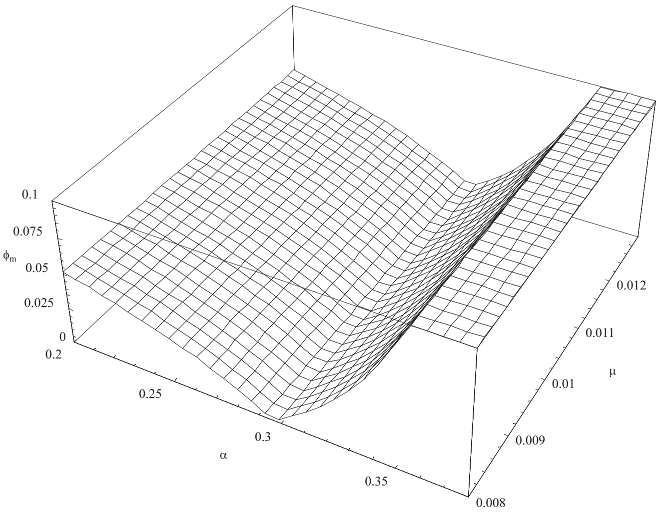

FIG. 5. Three-dimensional plots of the variation of the amplitude $|\phi_m|$ of the ion-acoustic double layers with plasma parameters (α and μ) having γ=0.1 and β=0.05.

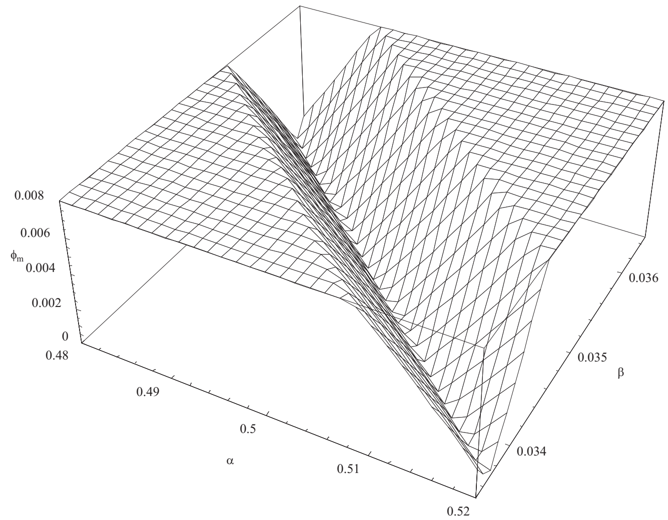

FIG. 6. Three-dimensional plots of the variation of the amplitude $|\phi_m|$ of the ion-acoustic double layers with plasma parameters (α and β) having γ=0.1 and μ=0.01.

eters with $\alpha < \alpha_R$, there exist ranges of positron concentration below and above the critical concentration α_Q, in which the system supports rarefactive and compressive double layers, respectively. Outside these ranges (below α_Q for rarefactive double layer and above α_Q for compressive double layers), we find that either $|\phi_m| \geqslant 1$ or $Q \neq \mathrm{O}(\varepsilon)$ and hence the theory for weak double layers is no longer valid. From Fig. 1 it may also be noted that the effect of finite positron temperature is to decrease (increase) the amplitude of the rarefactive (compressive) double layer and increase (decrease) its width.

In Fig. 2 curves show the variation of the amplitude and width of the ion-acoustic double layers with positron concentration for fixed values of parameters (β and γ) with three different values of cold electron concentration (μ). These curves show that for the fixed values of the parameters, on increasing the cold electron concentration (μ) the existence region of the double layers shifted to the higher positron concentration side. The effect of the temperature ratio of cold to hot electrons (β) on the existence regions and on the characteristics of the double layers is shown in Fig. 3.

Figure 4 shows the three-dimensional plots of the variation of the amplitude of the ion-acoustic double layers as a function of plasma parameters, i.e., positron concentration (α) and positron temperature (γ). In Fig. 5, we have shown the variation of the amplitude of the ion-acoustic double layers as a function of positron concentration (α) and cold electron concentration (μ), whereas Fig. 6 represents the variation of the amplitude of the ion-acoustic double layers as a function of positron concentration (α) and the temperature ratio of cold to hot electrons (β).

VI. CONCLUSIONS

To summarize, we have investigated the ion-acoustic double layers in electron-positron ion plasmas. It is found that there exist two critical concentrations of positrons, α_R and α_Q, which decide the existence and nature of the ion-acoustic double layers. It is also found that the system supports ion-acoustic double layers only when the positron concentration (α) is less than the critical concentration α_R (i.e., $\alpha < \alpha_R$). The present theory also predicts that for the given set of parameter values, if $\alpha_R < \alpha_Q$ the system supports only rarefactive double layers for the values of α lying in the range $0 < \alpha < \alpha_R$. However, for the given set of parameters (μ, β, and γ), if $\alpha_R > \alpha_Q$, the system supports rarefactive double layers for $\alpha < \alpha_Q$ and for $\alpha > \alpha_Q$, compressive double layers exist. It is also investigated that for a given set of parameter values on increasing the positron concentration, the amplitude of the rarefactive (compressive) double layer decreases (increases), whereas as positron concentration is increased, the width of the rarefactive (compressive) double layer increases (decreases). The effect of finite positron temperature is to decrease (increase) the amplitude of the rarefactive (compressive) double layer and increase (decrease) its widths.

The present investigation predicts findings on the existence of the small amplitude ion-acoustic double layers in the presence of cold ions, hot positrons, and two temperature distributions of electrons. In actual situations, ion-acoustic wave events associated with positrons are frequently observed in the solar atmospheres. Hence, referring to the present studies, we think that these results will be helpful in understanding double layers associated with the ion-acoustic waves in space where the positrons exist. Although we have not referred to any specific observations, the present theory is applicable to analyzing small amplitude ion-acoustic double layers associated with positrons which may occur in space, active galactic nuclei, and solar atmosphere.

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